Brownian Motion and Exposed Solutions of Differential Inclusions

Vladimir V. Goncharov
CIMA-UE, Universidade de Évora, Portugal
goncha@uevora.pt

April, 24, 2013

Abstract

We present a new method for proving the existence of (Carathéodory) solutions to differential inclusions based on the properties of uniformly distributed *Brownian motion* and dual in some sense to the famous *Baire category approach*. Namely, considering an (autonomous) differential inclusion

$$\dot{x} \in F(x), \tag{1}$$

where $F : \mathbb{R}^N \rightrightarrows \mathbb{R}^N$ is a bounded Hölder continuous (with an exponent $\alpha > 1/2$) multifunction with compact and convex values, by using our method we establish existence of a solution $x(\cdot)$, $x(0) = x_0$, to the inclusion (1) such that the derivative $\dot{x}(t)$ is almost everywhere an *exposed point* of the right-hand side of (1). Besides that, in the case when $F(x) = [f(x), g(x)]$ is a nondegenerate interval moving in the lipschitzean way, we construct a canonical probability measure supported on the solution set and such that *almost surely* (in the sense of this measure) the derivative $\dot{x}(t)$ is equal either to $f(x(t))$ or to $g(x(t))$ for a.e. t. The research is originated by an idea of A.Bressan and fulfilled jointly with G.Colombo in a particular case $N = 2$ (it is well-known that the set of exposed points of a compact convex set $A \subset \mathbb{R}^N$ is *strictly* included into the set of extreme ones already in this simplest case).