СЕМИНАР ОДС 12 октября 2016 года

Актовый зал 15:00

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From games to fixpoints and backward. I

1. There is a fairly direct connection between the common fixpoints and the differential games: the key object in the solution of the differential game of quality — the maximal stable bridge[1] — is the greatest fixed point of programmed absorption operator, that self–maps the boolean of the system positions[2]. The action of this operator is the result of actions of some operator family, such that common fixpoints of the family are fixpoints of the operator. It makes possible to describe the object of interest — the maximal stable bridge — in terms of the operator family with a relatively simple structure.

The first of provided results develops in a constructive direction well–known Tarski's theorem[3] on the structure of the set of common fixed points of self–maps on a complete lattice. This theorem has a number of generalizations[4][5][6] weakening demands on the order structure and upgrading in an appropriate manner the assertion on the structure of the set of common fixed points. However, there is a lack of statements like the Kantorovich theorem[7] or Kleene theorem that describe fixpoints in terms of convergent sequences of the operator degrees.

2. In view of the known connection between fixed points and equilibrium points, one would expect a fixed point result under conditions similar to the conditions of the famous Fan minimax theorem[8]. We recall that this theorem has a criterion character and uses nothing but the topological properties of the quality index domain and of the index itself.

In the second group of results, we give the description (redefinition) of the set of fixed points in the set-theoretic terms. This general definition is used to provide necessary and sufficient conditions for the existence of fixed points for cases where the set is endowed with Hausdorff topology and the mapping has closed graph. An example illustrating the possibilities and advantages of the proposed approach is given. The immediate application is given on the example of Nash equilibrium problem.

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[2]Chentsov A.G. On a game problem of converging at a given instant of time, *Mathematics of the USSR-Sbornik*, 1976, vol. 28, no. 3, pp. 353–376.

[3]Tarski A. A lattice-theoretical fixpoint theorem and its applications // Pacific Journal of Mathematics. 1955. Vol. 5, no. 2. pp. 285–309.

[4]Markowsky George. Chain–complete posets and directed sets with applications // algebra universalis. 1976. Vol. 6, no. 1. pp. 53–68.

[5]Cousot, P., Cousot, R. Constructive versions of Tarski's fixed point theorems // Pacific Journal of Mathematics, 1979, 82, no. 1., pp. 43–57.

[6]Ronse Christian. Lattice–theoretical fixpoint theorems in morphological image filtering // Journal of Mathematical Imaging and Vision. 1994. Vol. 4, no. 1. pp. 19–41.

[7]Kantorovitch L. The method of successive approximation for functional equations // Acta Mathematica. 1939. December. Vol. 71, no. 1. pp. 63–97.

[8]Fan K. Minimax theorems // Proc. Nat. Acad. Sci. U.S.A. 1953. V. 39. pp. 42–47.