Problem of vertical passing an obstacle by an aircraft under wind disturbances

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European Control Conference ’2009

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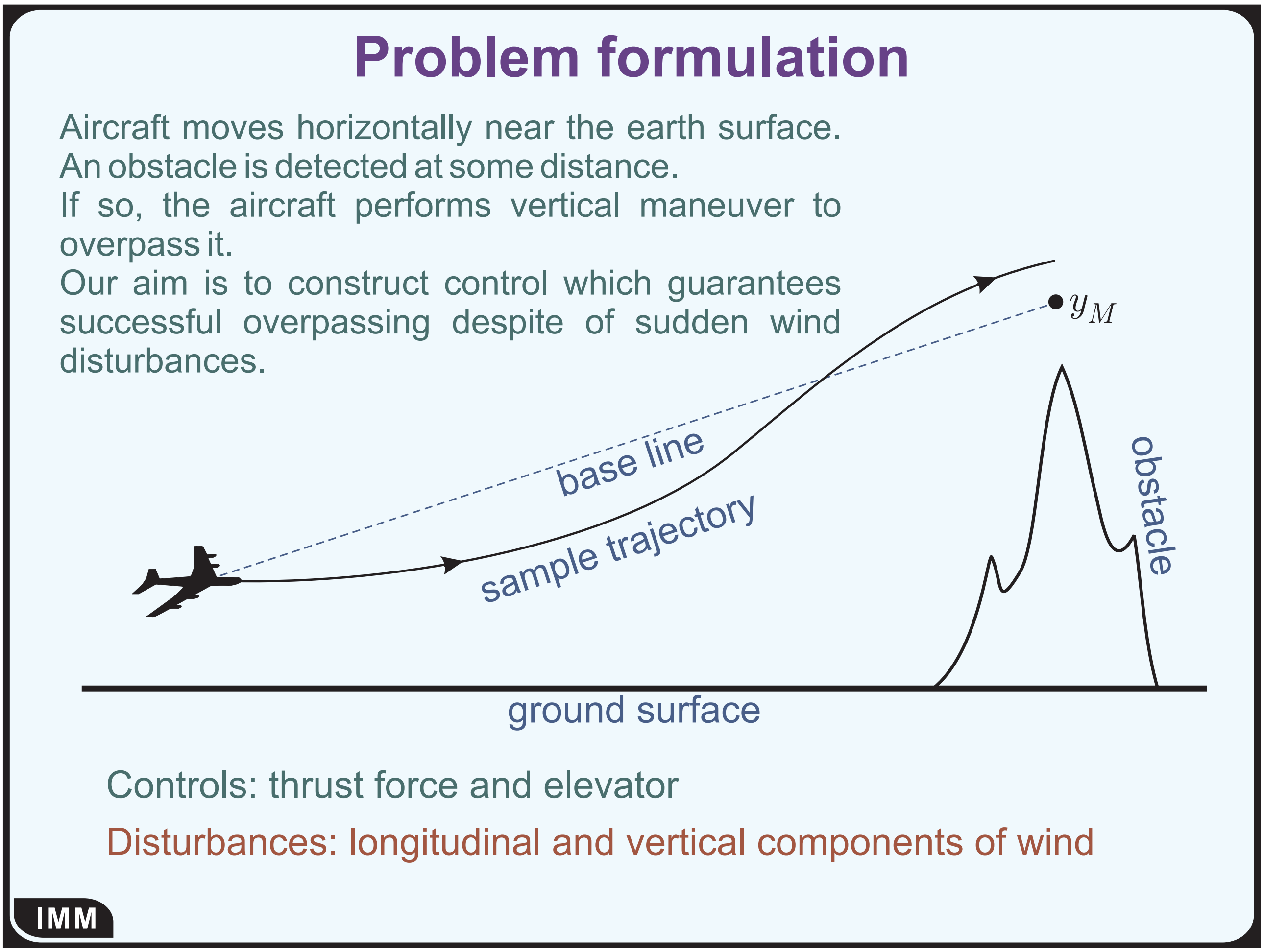
This work was presented in
European Control Conference ’2009
Budapest, Hungary, 23–26 August 2009

Link to this slides and corresponding paper:
`http://home.imm.uran.ru/sector3/ecc2009/`



Slide 1. We consider a problem of overpassing a vertical obstacle by a midsize aircraft.

Aircraft dynamics

$$\dot{x}_g = V_{xg}$$ – here, x_g is longitudinal coordinate

$$\dot{V}_{xg} = [(P\cos\sigma - qSc_x)\cos\vartheta - (P\sin\sigma + qSc_y)\sin\vartheta)]/m$$

$$\dot{y}_g = V_{yg}$$ – y_g is altitude

$$\dot{V}_{yg} = [(P\cos\sigma - qSc_x)\sin\vartheta + (P\sin\sigma + qSc_y)\cos\vartheta]/m - g$$

$$\dot{\vartheta} = \omega_z$$ – ϑ is pitch angle

$$\dot{\omega}_z = M_z/I_z$$ – ω_z is angular velocity

$$\dot{P} = k_p(\delta_{ps} - P)$$ – P, δ_{ps} are thrust force and its control

$$\dot{\delta}_e = k_e(\delta_{es} - \delta_e)$$ – δ_e, δ_{es} are elevator deflection and its control

Introduce a new control u_ϑ – the command pitch angle:

$$\delta_{es} = k_1(\vartheta - u_\vartheta) + k_2(\widehat{V} - \widehat{V}_0) + k_3\omega_z$$

Constraints:

$$0^\circ \le u_\vartheta \le 20^\circ$$

$$P_0 \le \delta_{ps} \le P_0 + 1.2m,$$

$$\boxed{\begin{aligned}|\Delta w_{xg}| &\le 10 \text{ m/sec},\\ |\Delta w_{yg}| &\le 5 \text{ m/sec}\end{aligned}}$$

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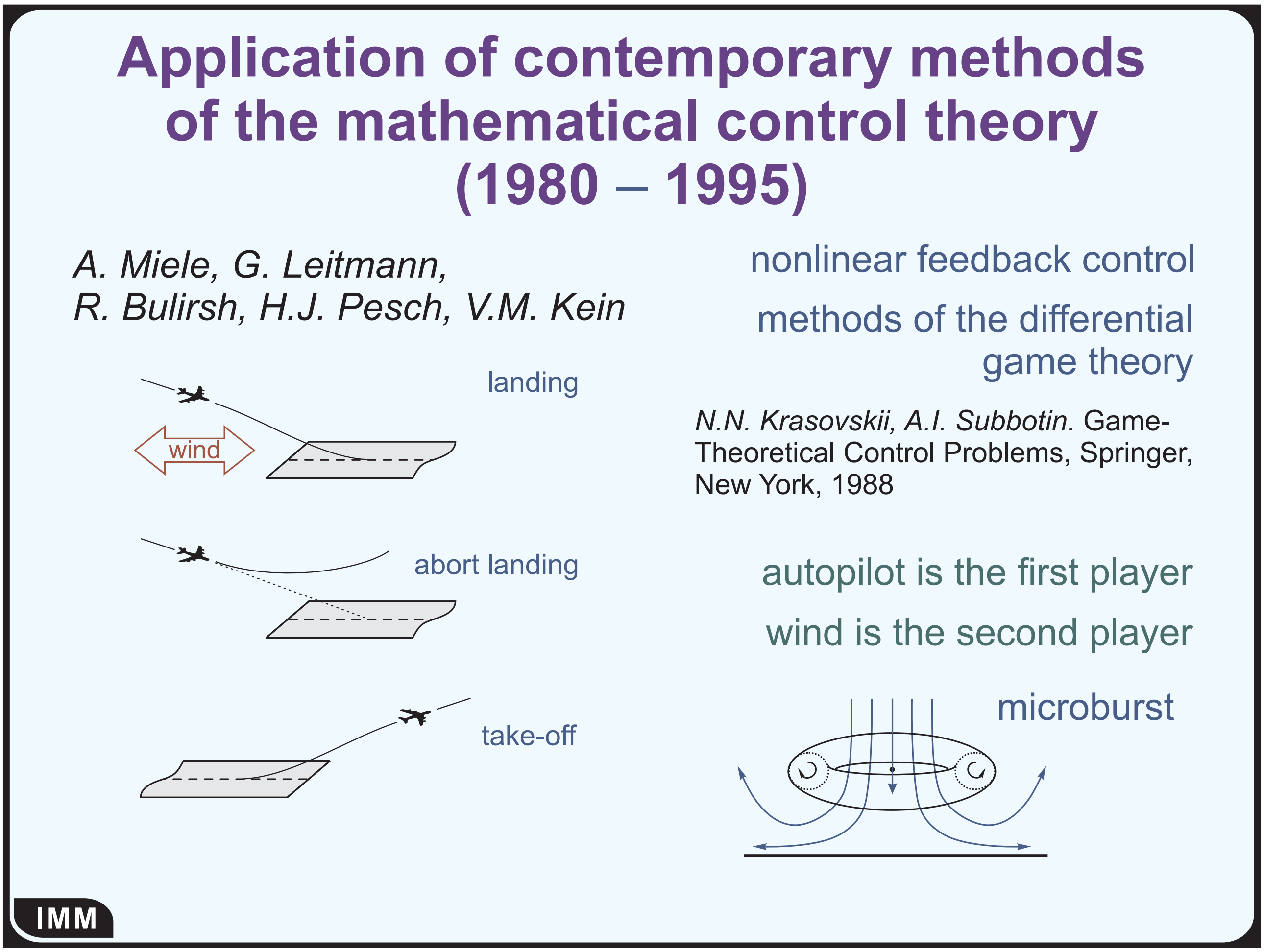
Slide 2. These are nonlinear differential equations of aircraft motion in the vertical plane. Standard Russian denotations are used. We need to take into account the state constraint on pitch angle. Therefore, we introduce a new fictive control, namely, "command pitch angle", on which the necessary constraint is put. The elevator control is defined by the linear law. Here, $\widehat{V}$ is the relative velocity, and $\widehat{V}_0$ is its nominal value. The symbol m in the constraint on the thrust force command control is aircraft mass. We do not know what a wind will be. But we can take a reasonable constraint on deviation Δw_{xg} and Δw_{yg} of its components from some nominal values.



Slide 3. These are the names of scientists who applied contemporary methods of nonlinear feedback control to aircraft landing, abort landing, and take-off in the presence of wind disturbances. First of all, A. Miele is. His statements of such problems and mathematical models are very popular. In particular, G. Leitmann, R. Bulirsh, and H. Pesch tested their algorithms using the Miele's statements. Russian engineer and mathematician V. Kein was the first who applied the differential game theory methods (namely, the methods elaborated by N. Krasovskii and A. Subbotin) to aircraft motions. In computer simulations, different models of wind microburst are used. An air flow strikes the ground and flaws with creating the whirles.

Adaptive control

$$\dot{\mathbf{x}} = A(t)\mathbf{x} + B(t)u + C(t)v,$$
$$\mathbf{x} \in R^m, \quad t \in T = [t_*, t^*],$$
$$u \in \mathcal{P} \subset R^p, \quad v \in k\mathcal{Q}_{\max} \subset R^q,$$
$$\mathbf{x}|_n(t^*) \in M \subset R^n$$

$\mathcal{Q}_{\max}$ is critical (reasonable) constraint on disturbance

k is *disturbance level* – unknown in advance

Requirements on *adaptive* control $U(t, \mathbf{x})$:

1) if $k = 1$, then control should lead the system to the target set M despite of the actions of the disturbance;

2) if $k < 1$, then the leading should be done by a control constrained by $\mathcal{P}_k$, which is less than $\mathcal{P}$; when k becomes smaller, $\mathcal{P}_k$ becomes smaller too;

3) if $k > 1$, the terminal miss is allowed, but it should be estimated from above.

These requirements are used for zero initial position. If initial point is non-zero, then some level of control is used to compensate this deviation.

S.A. Ganebny, S.S. Kumkov, V.S. Patsko, 2005 – 2008

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Slide 4. Now, our method of adaptive control is elaborated for linear dynamic systems. A control vector u of the first player is restricted by geometric constraint $\mathcal{P}$. We postulate some set $\mathcal{Q}_{\max}$ as a critical (reasonable) constraint on control vector v of the second player (disturbance), but actually we do not know nonnegative scalar coefficient k in advance. The first player tries to lead some n coordinates of the state vector $\mathbf{x}$ to the target set M at the terminal instant t^*. The main peculiarities of our method are explained via properties 1)–3).

Transformation to reduced dynamics

Linearization of dynamics with respect to the base trajectory:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu + Cv, \quad \mathbf{x} \in R^8, \quad t \in [t_*, t^*], \quad u \in \mathcal{P}, \quad v \in k\mathcal{Q}_{\max}$$

$\mathbf{x}$ is the vector of relative coordinates,

$$u = \begin{pmatrix} \delta_{ps} - P_0 \\ u_\vartheta - \vartheta_0 \end{pmatrix}, \qquad v = \begin{pmatrix} w_{xg} - w_{xg0} \\ w_{yg} - w_{yg0} \end{pmatrix}$$

Transformation to reduced dynamics:

$$\dot{\xi} = D(t)\delta + E(t)w, \quad \xi \in R^1, \quad t \in [t_*, t^*], \quad u \in \mathcal{P}, \quad v \in k\mathcal{Q}_{\max}$$

ξ is predicted altitude deviation at the final instant t^* if zero controls are applied to the system

Transformation formulas:

$$\xi(t) = \Phi_3(t^*, t)\mathbf{x}(t), \quad D(t) = \Phi_3(t^*, t)B, \quad E(t) = \Phi_3(t^*, t)C$$

$\Phi_3(t^*, t)$ is the third row of the Cauchy matrix for $\dot{\mathbf{x}} = A\mathbf{x}$

J. Shinar, V. Glizer, V. Turetsky, S. Le Menec

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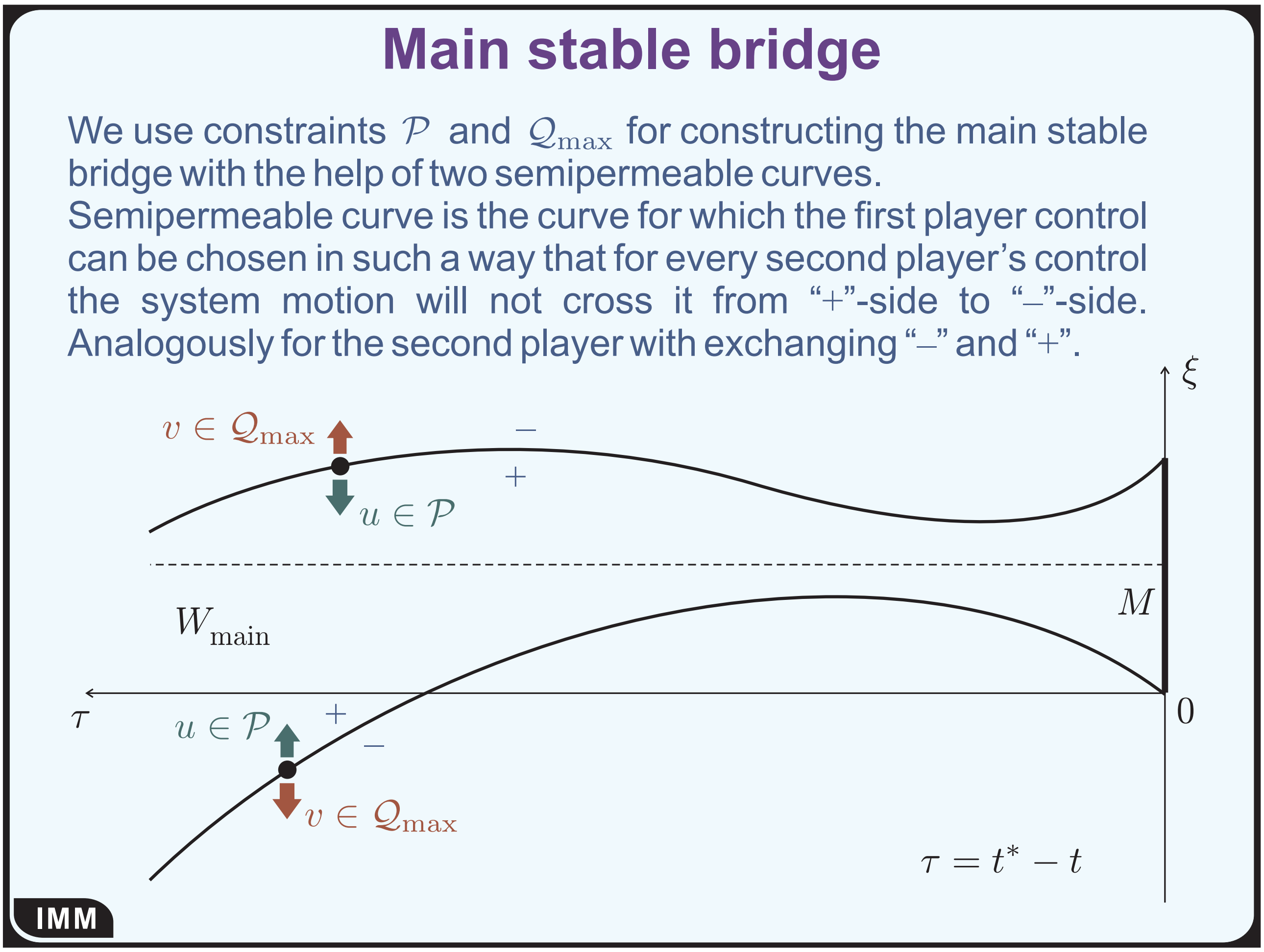
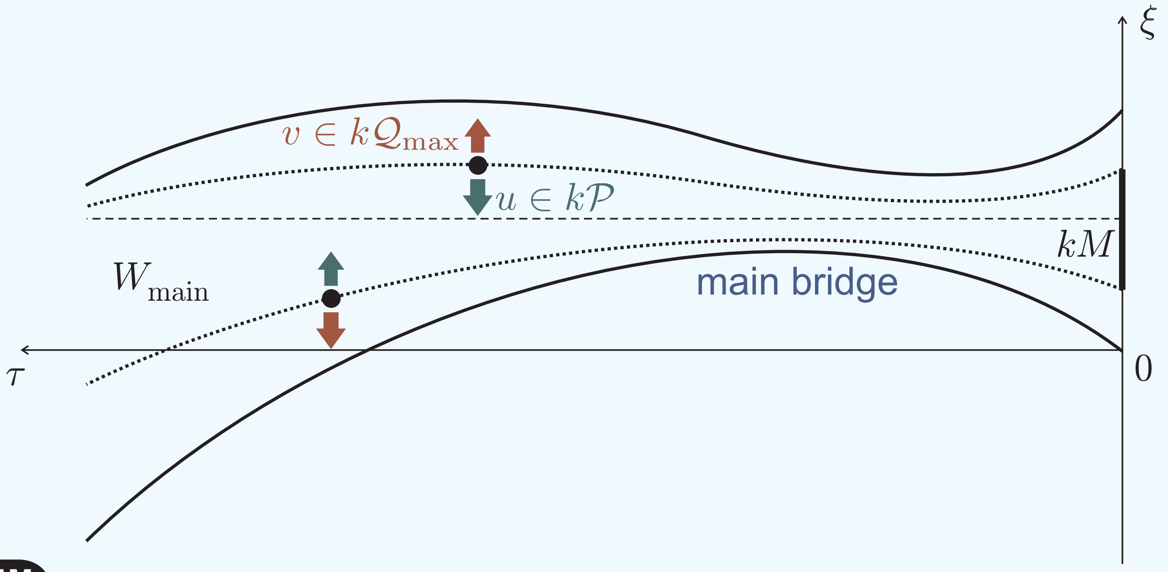
Slide 5. So, we linearize the aircraft dynamics with respect to the base rectilinear trajectory. Here, t^* is the prognosis instant of obstacle overpassing. We are interested only in the vertical coordinate at the instant t^*. Therefore, we use the standard transformation to one-dimensional dynamics, where state coordinate ξ is forecast altitude deviation from the nominal value at the instant t^*. J. Shinar and his coworkers V. Glizer and V. Turetsky used very often such a transformation in their investigations. Also, S. Le Menec does.



Slide 6. The main subject in our adaptive control is the construction of the main stable bridge W_{main}. We consider the auxiliary differential game with one-dimensional state variable ξ, backward time τ, and the target set M. The target set is a tolerance on deviation of vertical coordinate at the nominal instant of overpassing the obstacle. The main bridge W_{main} is the solvability set, from which the first player guarantees leading the system to the set M despite of the actions of the second player taken from the constraint $\mathcal{Q}_{\max}$. The constuction of the main bridge W_{main} is very simple because his boundaries are semipermeable curves. We choose some inside "central" line for constructing adaptive control.

Adaptive control idea

The set computed with multiplication of the main stable bridge by a scalar $k \geq 0$ in each τ-section is the stable bridge with constraints on control and disturbance multiplied by k, and the target set changed in the same way. This new bridge is inside the main bridge.



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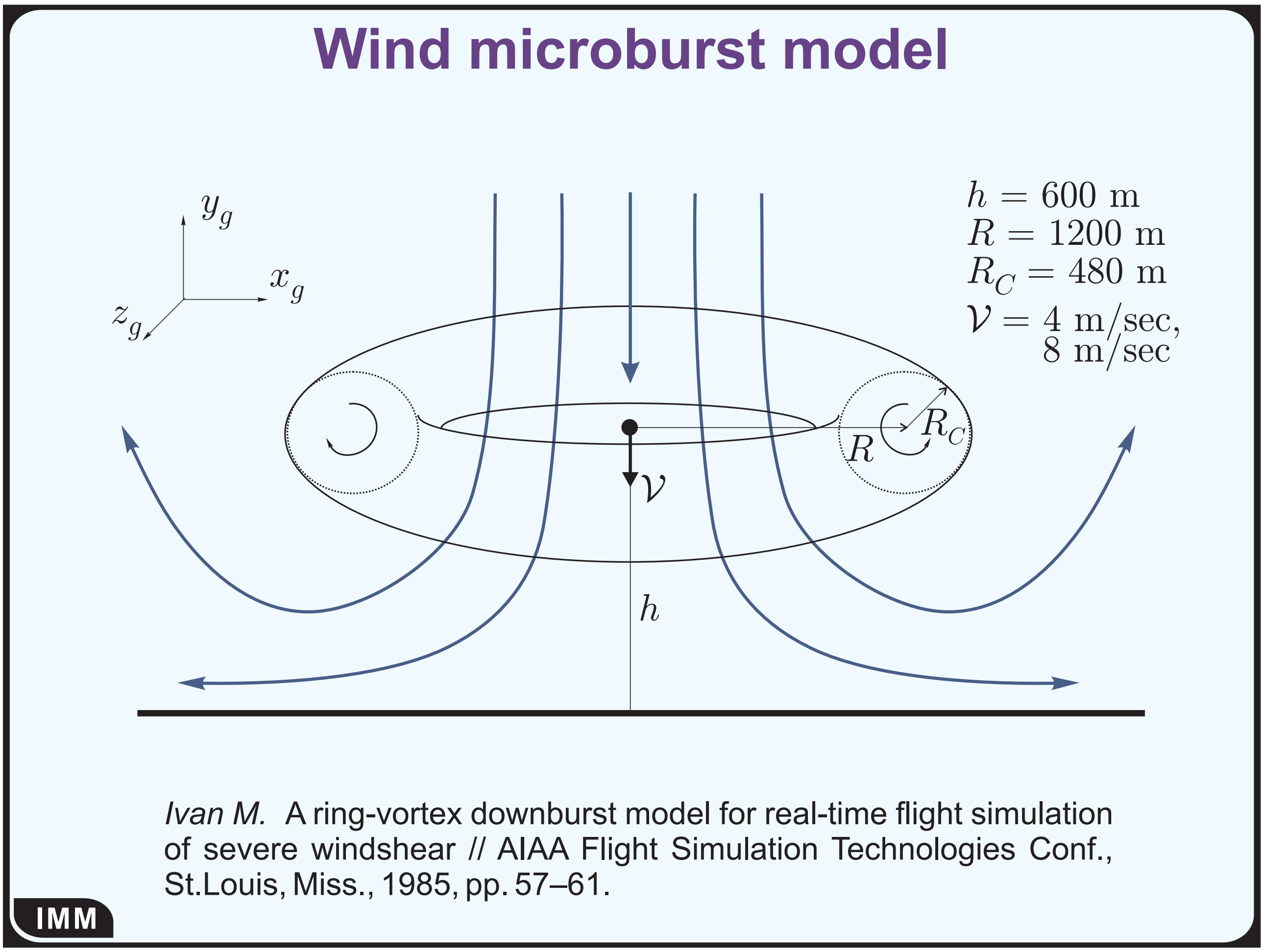
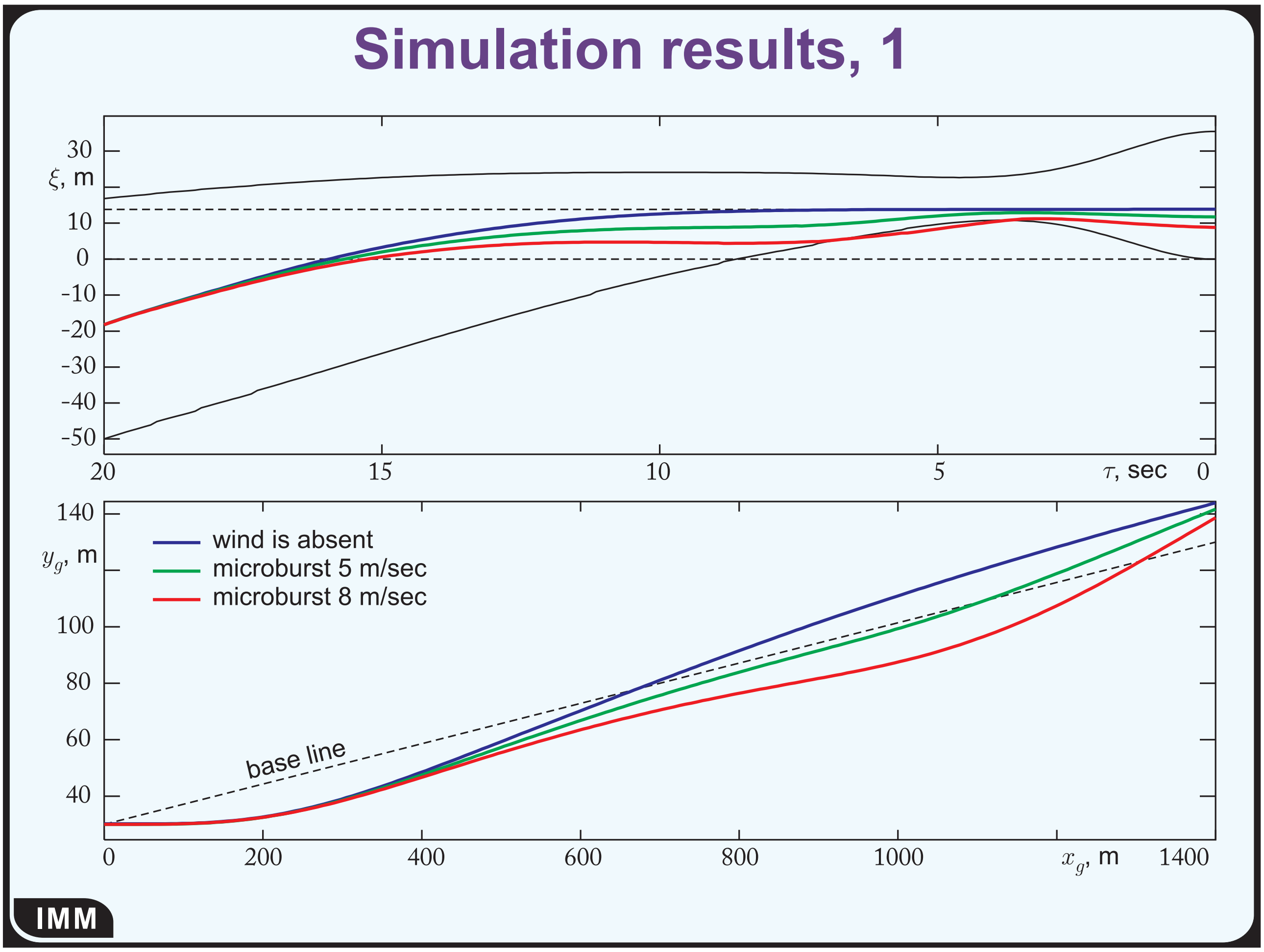
Slide 7. The adaptive feedback control inside the set W_{main} is implemented on the base of compression this set with respect to the central line in each τ-section. If a current state variable ξ is inside the main bridge, we determine the coefficient k ($k = 0$ on the central line) and take the extreme control u from the set $k\mathcal{P}$. So, if k is small, then the level of the control is small also. Outside the set W_{main}, we take the control u from the set $\mathcal{P}$.



Slide 8. There are several microburst models in scientific literature. Very simple but non-trivial microburst model was elaborated by M. Ivan. We use this model for generating the wind velocity. The microburst geometry is given by four parameters. The values taken are shown in the right. The wind velocity at the central point equals 4 m/sec for “weak” microburst and 8 m/sec for “strong” one.



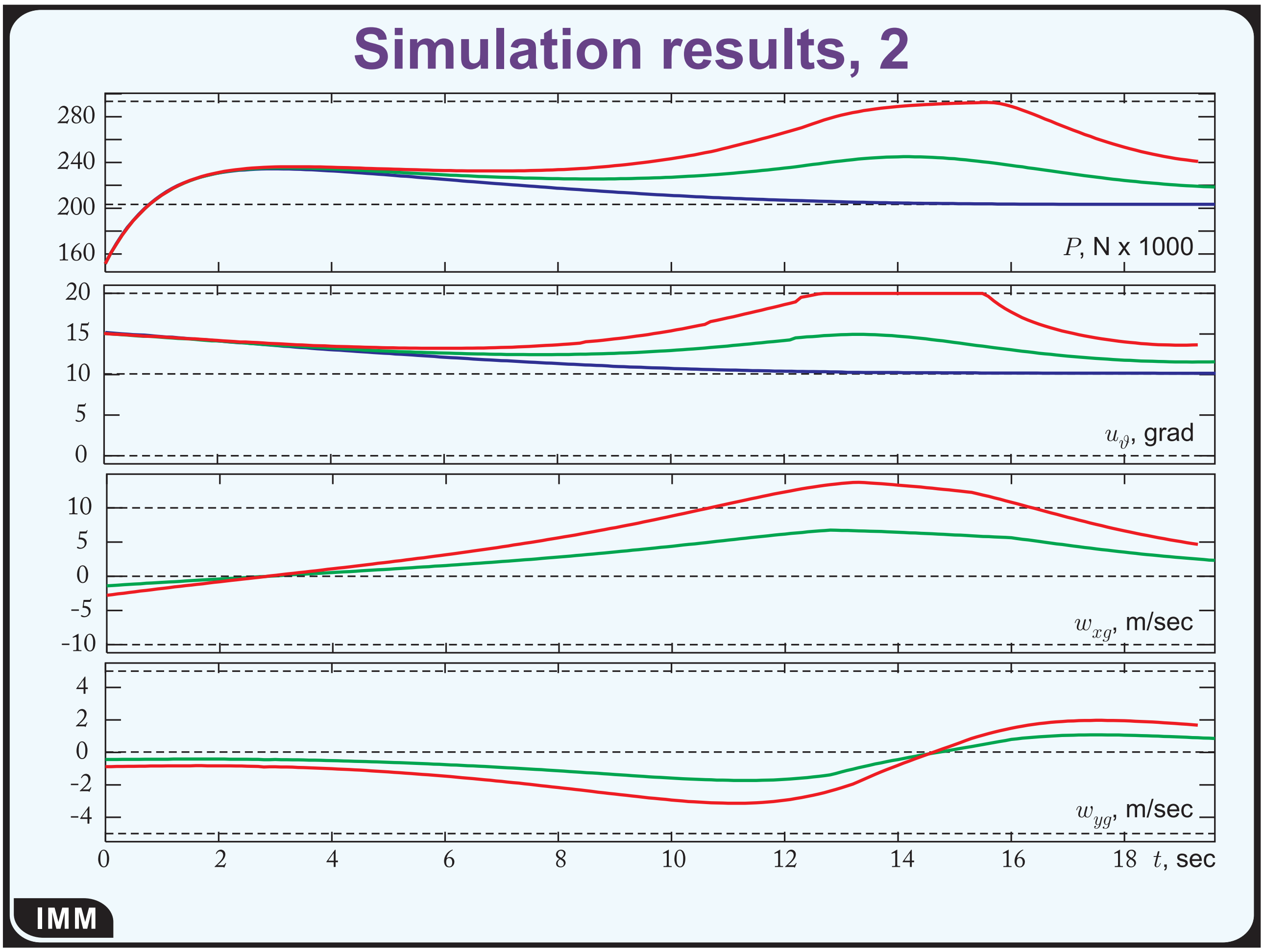
Slide 9. Let suppose that nominal values of wind velocity components are equal to zero.

We assume that during overpassing the obstacle the aircraft crosses a wind microburst zone. The microburst is symmetrically placed with respect to the projection of the base line onto the ground plane. Thus, the wind disturbance acts only in the vertical plane. The microburst central point is situated on x_g-axis at the distance 200 m from the initial position of overpassing.

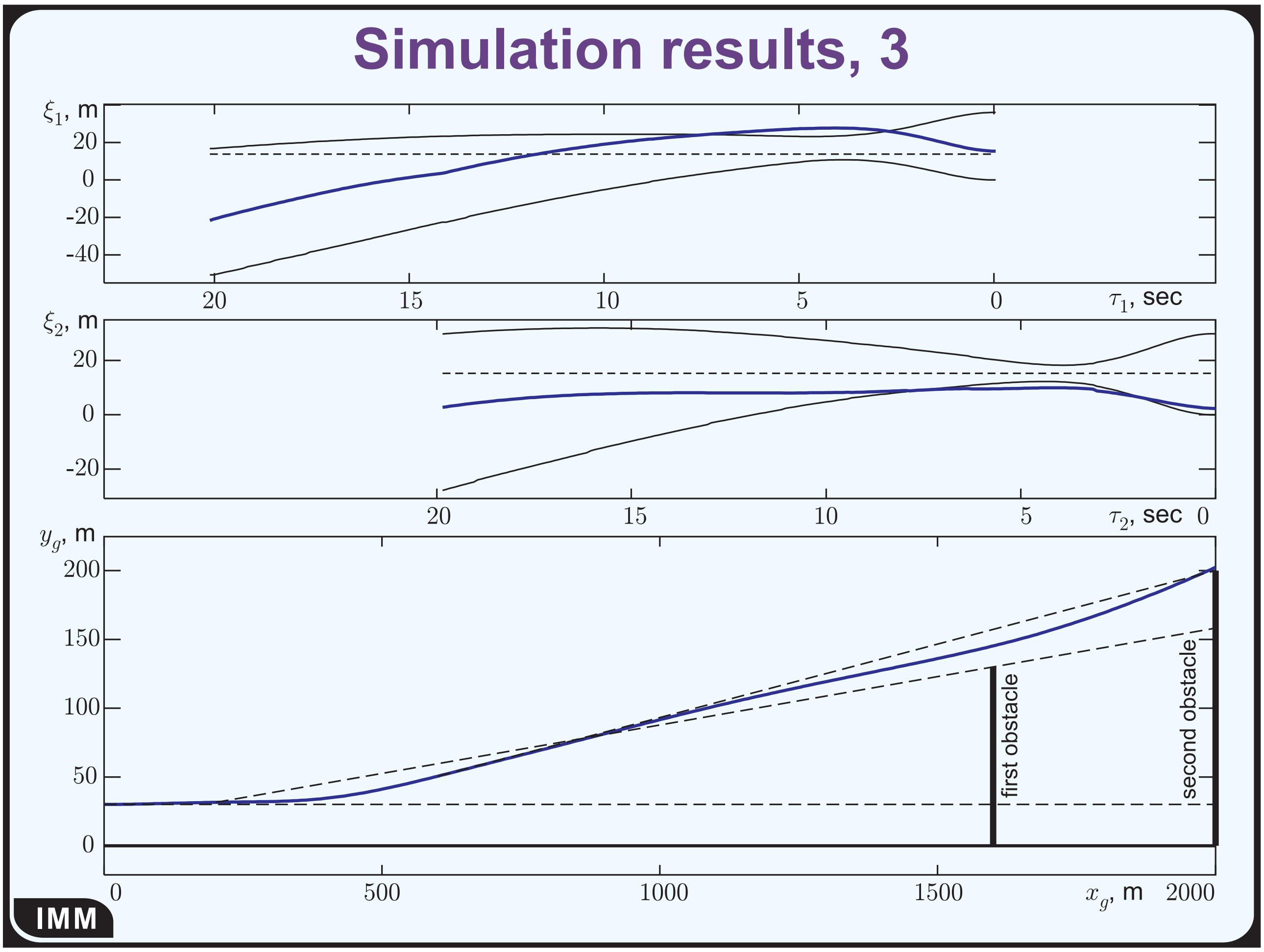
We compute the motions of the nonlinear aircraft system. With that, the adaptive feedback control is elaborated on the basis of one-dimensional differential game. Three cases are considered: wind is absent, wind is generated by weak microburst, and wind corresponds to stong microburst.

In the upper picture, the trajectory curves in the space $\tau \times \xi$ of the one-dimensional game are shown.

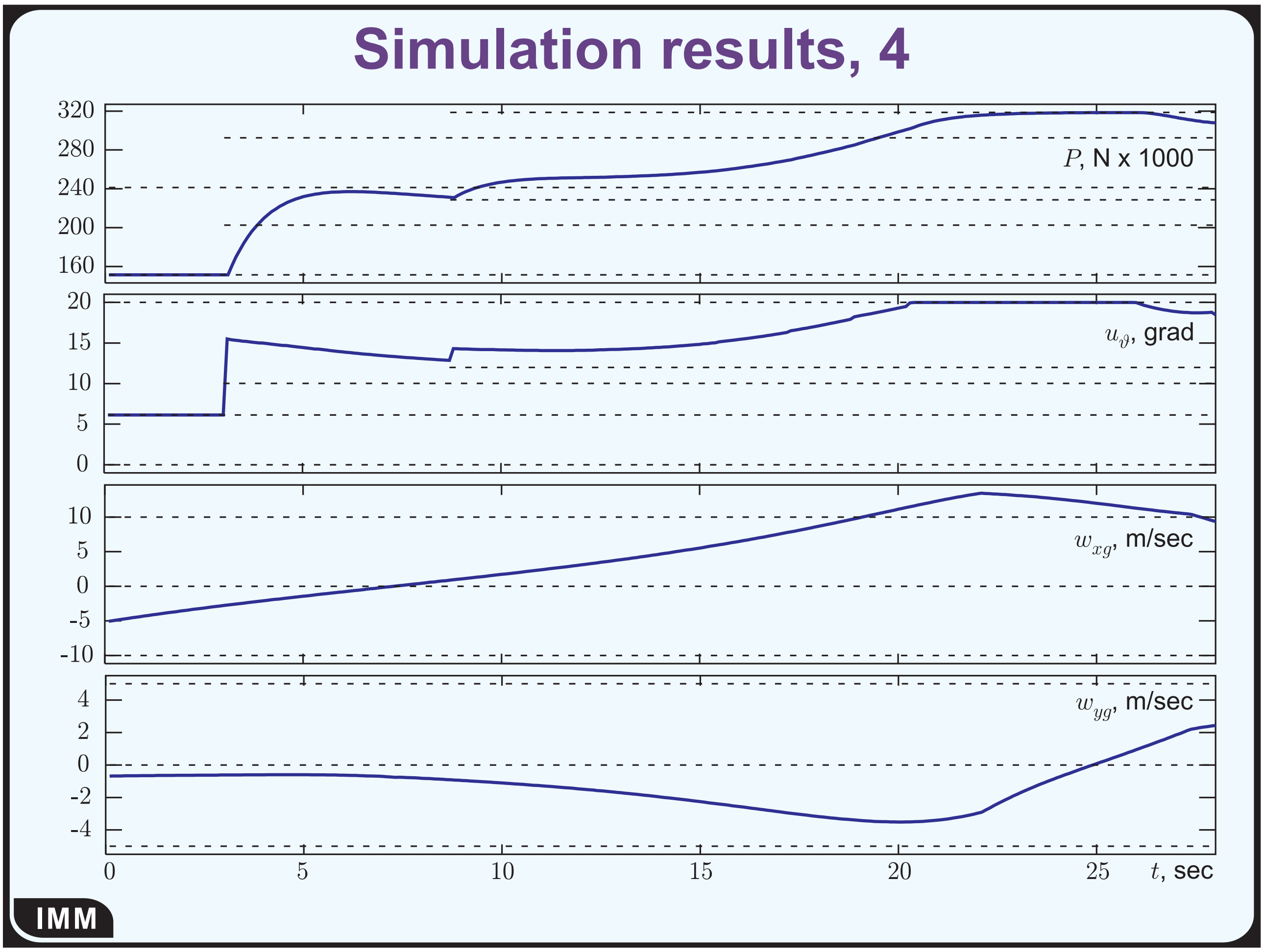
In the lower picture, we see the corresponding trajectories in the vertical plane.



Slide 10. Here, the processes of the thrust force, command pitch angle, and wind components are shown. The controls on P and u_ϑ reach their boundary values only in the case of strong microburst, namely, in the time interval 12.5–13.5 sec when the wind disturbance component w_{xg} is large.



Slide 11. Now, we consider a situation when the second obstacle is detected after some time of overpassing the first obstacle. So, two one-dimensional differential games are analysed. Each of them is considered from the distance 1400 m up to the obstacle. In this slide, the implementations of trajectories are shown for the case of strong microburst, the center of which is placed on the x_g-axis at the distance 500 m from the beginning.



Slide 12. Here, the other realizations concerning the case of two obstacles and strong microburst are given.