Reachable Sets for Some Simple Models of the Car's Motion

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Mathematical cars

Dubins’ car
\[
\begin{align*}
\dot{x} &= \sin \theta \\
\dot{y} &= \cos \theta \\
\dot{\theta} &= u, \quad |u| \leq 1
\end{align*}
\]


Reeds-Shepp’s car
\[
\begin{align*}
\dot{x} &= w \sin \theta \\
\dot{y} &= w \cos \theta \\
\dot{\theta} &= u, \quad |u| \leq 1, \quad |w| \leq 1
\end{align*}
\]


Given-time reachable sets in the plane of geometric coordinates (Dubins´ car)

\[ T = 0.5\pi \]

\[ T = \pi \]

\[ T = 1.5\pi \]

\[ T = 2\pi \]

Time-limited reachable sets

\[ T = 0.5\pi \]

\[ T = \pi \]

\[ T = 1.5\pi \]

\[ T = 2\pi \]
Given-time reachable sets (Reeds-Shepp’s car)

\[ T = 0.127\pi \]

\[ T = 0.51\pi \]

From Dubins’ car to Reeds-Shepp’s car

\[
\begin{align*}
\dot{x} &= w \sin \theta \\
\dot{y} &= w \cos \theta \\
\dot{\theta} &= u
\end{align*}
\]

\[|u| \leq 1, \quad a \leq w \leq 1\]

\(a\) - parameter

Dependence of reachable sets on parameter \(a\) ?
Isaacs’ transformation

\[ \dot{x} = -y \, u \]
\[ \dot{y} = x \, u - w \]
\[ |u| \leq 1, \quad a \leq w \leq 1 \]

Reachable set of the original system is the collection of points from which this system can be brought to the origin at a given time or by a given time.
Backward construction

$F_3$

$W(3\Delta, M)$

$F_2$

$W(2\Delta, M)$

$F_1$

$W(\Delta, M)$

$\Gamma_0$

$M$
Given-time reachable sets for different values of $a$

$T = 0.9$

$T = 1.8$
Time-limited reachable sets for different values of $a$

$T = 0.9$

$T = 1.8$
Reachable sets corrected by accounting for oriented added shift

Collection of points in the plane $x, y$ which can be covered by given orientedly added set at time $T$
Reachable sets for oriented shift \((-1.2, 0)\)

\[ a = -0.6, \ T = 2.6, \ \Delta = 0.002, \ \delta = 0.04 \]
Reachable sets for oriented shift \((-0.5, -0.5)\)

\begin{align*}
a &= 0, & T &= 2.4 \\
\Delta &= 0.002, & \delta &= 0.04
\end{align*}
Time-limited reachable sets for oriented shift \((-0.5, -0.5)\)

\[ a = 0.8, \ T = 4.75 \]
\[ \Delta = 0.002, \ \delta = 0.04 \]
Value function

$a = 0$

$a = 0.8$
Three-dimensional given-time reachable set for Dubins’ car

\[ T = \pi \]
Time development of reachable sets for Dubins’ car

$T = \pi$

$T = 2\pi$

$T = 3\pi$

$T = 4\pi$
Violation of simple connectedness of reachable set

\[ T = 3.63\pi \]
\[ T = 3\pi \]
Given-time reachable sets for angle $\theta$ modulo $2\pi$
Reachable sets in cylindrical coordinates

\[ T = 1.5\pi \]

\[ T = 2\pi \]
Authors´ works


