To memory of Lawrence Shepp

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Working on probability theory and mathematical statistics, far away from optimal control theory, in 1990, Larry Shepp surprisingly published a joint paper with Jim Reeds on a model of control motion in the plane. In this model, an object going forward can stop immediately and continue to move in the opposite direction. The model became very popular and received the name “Reeds and Shepp’s car”. Now it is intensively used in theoretical robotics and mathematical control theory.

We also use this model in works related to differential games.

1. Introduction. We want to find a shortest path in the plane with specified initial and final points and directions and with the further constraint that at each point the radius of curvature should be \( \geq 1 \).

This problem arose in a simple model for a robot cart which moves under computer control. The cart can shift into reverse and so the path is allowed to have cusps.

In an elegant paper, Lester Dubins (1957) solved the problem when the car cannot reverse and cusps are not allowed. Even in this case it is apparently impossible to give an explicit formula for the shortest path. Instead Dubins gives a sufficient set of paths, i.e. a set which always contains what he called a geodesic, or optimal path. His sufficient set is so small that there are at most 6 contenders in the set for each case of specified endpoint conditions, and it is a simple matter to find the shortest of these 6, which gives an algorithm for the solution. He showed that any geodesic can be described by one of 6 words: \( \pi \rho / \pi \),

\begin{align*}
\ell \quad &\frac{3\pi}{2} \quad S_2 \quad \frac{3\pi}{2} \\
\text{FIGURE A}
\end{align*}
From August 1966 to January 1967 Larry Shepp had a fellowship in the Kolmogorov group at the Moscow State University. Larry knew that we work on differential games and described his stay in Russia with playful words (letter to us, 14.03.2009):

„By the way, I knew Pontryagin very well. I lived in Moscow from August, 1966 to January, 1967. I speak Russian which I studied only as an adult. I was forced to leave early due to a pursuit game I got into with the KGB playing the role of the pursuer and me playing the role of the evader: I arrived at the US Embassy just ahead of their attempts to grab me. The entire game took place in front of the embassy. The game was a variant where the evader has to get into the door rather than just avoid the tag for a maximal time. Clearly neither of us was using an optimal solution; I survived for nearly 50 years, so the evader seems to have won - I ran faster in those days than I do today, and they were not expecting me, apparently, or chose not to catch me due to other criteria entering. “

Since 1989, Larry has visited Russia many times. He used to attend mathematical conferences and gained many friends in Russia.
Mathematical flower to memory of Larry Shepp

\[ \dot{x}_p = w \sin \theta \]
\[ \dot{y}_p = w \cos \theta \]
\[ \dot{\theta} = u, \ |u| \leq 1, \ a \leq w \leq 1 \]

Graph of the minimum-time function for \( a = 0.8 \) and terminal point \( m \)

- \( a = 1 \) : Dubin´s car
- \( a = -1 \) : Reeds-Shepp´s car