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GEOMETRIC MATCHING OF TRACKS FROM SEVERAL RADARS AND ESTIMATION OF SYSTEMATIC ERRORS IN MEASUREMENTS *

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Abstract

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A problem of processing the measurements from air surveillance radars is considered. The measurements are provided by observation of air traffic. This processing aims at matching the tracks of aircrafts obtained by various radars and estimation of their systematic errors. The error estimates are built in the form of a "collection of corrections" for radars measurements. For finding estimates, redundancy of observations with data from several radars is used. In the approach suggested, neither one of radars is regarded as the reference standard in principle. The systematic errors are defined by combined procession of measurements from "neighboring" radars with overlapping observation zones.

Introduction

Measurements from several surveillance radars (in the sequel, radars) are processed. The measurements are recorded from observation of air traffic in a zone of size up to 1000 kilometers. Each measurement contains the radar's number, number of the aircraft, its current geometric coordinates, altitude, and instant of measuring. It is assumed that the primary procession of the data has been already performed with corresponding identification and rejection of unreliable information.

At the initial step of the algorithms suggested, the preliminary estimation of difference components of the systematic errors is implemented on the basis of the radar pairs. Here, the problem of geometric matching is solved with the block-wise procession of the data. As the block information, the parts of tracks are taken on small time-intervals. The local collections of measurements from various radars are considered as geometric figures or as collections of points.

During procession of such data collections from various radar pairs participating in observation, the parameters of matching the tracks of one radar w.r.t. another one are found. The corresponding direction and value of the track shift for all paired radars are calculated. Also, the standard deviation (SDV) of measurements w.r.t. the calculated direction of motion is computed. The mentioned direction is characterized by the maximal variance of the spread in measurement points. In defining the difference parameters of pair-wise matching of the tracks, the instants of the measurements are not used. This peculiarity is crucial since it allows one to determine systematic errors of aircraft geometric positions in the radar measurements without errors of joining the measurements instants to the global time-scale of the air traffic control (ATC) system. Additionally, after compensation of the geometric systematic errors, an opportunity appears to estimate the level of mistiming in the measurements from various radars.

The geometric position errors are decomposed into the error components in azimuth and in range. Corresponding resultant corrections for measurements are formed by processing the totality of local parameters of geometric matching the tracks from radars over the whole ATC zone. Here, a special optimization procedure is performed that minimizes the dissipation SDV between the calculated and actual matching parameters of tracks from various radars. The actual values are formed at the initial stage during the local procession of the data.

Geometric method for obtaining the systematic errors in the radar measurements

In the suggested method for finding estimates of the radar systematic errors, the geometric position errors and time-errors (the lag errors) are calculated independently. It is provided by preliminary geometric matching of the time-overlapping parts of tracks with using a collection of aircraft trajectories in some ATC zone.

For the geometric matching of the track parts from two and more radars, the method is based on constructing the "mean" line that approximates the aircraft direction of motion. The calculated data are taken from small geographic regions of the size up to 30–50 kilometers. This allows one to regard the measurements shifts (stipulated by the radar systematic errors) to be approximately constant vectors. Under this, it becomes possible to use only the shifts of tracks without deformation of the latter. The mentioned mean line is calculated by the principal component analysis [1] using the current fragment of motion.

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Calculations are performed in a local geographic coordinate system. Here, the axis OX is directed to the North, the axis OY is directed along the local vertical to the plane of the local horizon, the axis OZ is directed to the East. The system is tied to the point that is average over all measurements of all radars in the fragment of motion under consideration. The matching parameters obtained and used in the further procession are also joined to this point. For finding the estimate of the motion direction, the collections of measurements (from the radars participating in observation) are matched by the corresponding mass centers (the mathematical mean value of the geometric coordinates) of tracks. Further, using the obtained collection of measurements, the direction of the mean line is calculated by means of the principal component method. This direction provides the minimal SDV of all measurements w.r.t. the mean line.

We now present the necessary calculations for the plane case (without taking into account the aircraft altitudes). Let a centered collection of measurements $\{x_i, z_i\}$ of number n be given and $\{x_c, z_c\}$ be the central point. Finding the mean line, we calculate its inclination angle A_c relative to the axis OX counted clock-wise:

$$A_c = \frac{1}{2}\arctan\left(\frac{\sigma_{11} - \sigma_{22}}{2\sigma_{12}}\right).$$

Here, σ_{ij} are the elements of the covariance matrix

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \frac{1}{n}\begin{bmatrix} \sum_{i=1}^{n}(x_i - x_c)^2 & \sum_{i=1}^{n}\left((x_i - x_c)\cdot(z_i - z_c)\right) \\ \sum_{i=1}^{n}\left((x_i - x_c)\cdot(z_i - z_c)\right) & \sum_{i=1}^{n}(z_i - z_c)^2 \end{bmatrix}.$$

For all radars participating in observation over the considered part of the aircrafts motion, the shift vectors Sd_i are determined. These vectors are shifts of the central points of the radar tracks w.r.t. the mean line. Further, using these vectors for all possible radar pairs with numbers i, j ($i < j$), special values D_{ij} are calculated. These values are the relative geometric divergence (deviation) of tracks in the direction perpendicular to the mean line. The direction of this line is given by the vector $(\cos A_c, \sin A_c)$ in the local horizontal plane. The values D_{ij} are calculated as the scalar product $D_{ij} = \langle Sd_i - Sd_j, (\sin A_c, -\cos A_c)\rangle$.

Constructions in the plane are shown in Fig. 1. Here, using collections of measurements from two radars, the mean line is calculated that is determined by the current motion direction. The arrows show the deviation vectors Sd_1, Sd_2 of the corresponding central points of the tracks (outer circles) w.r.t. the mean line. The value D_{12} of the relative matching of these tracks is calculated as a residual of the vectors $Sd_1 - Sd_2$. Together with the direction of the mean line, this value goes into the further processing. In Fig. 1, the mean line (in dashes) passes through the "center-of-mass point" (inner circle) of the measurements, and the initial matched tracks from radars are also displaced towards the same point.

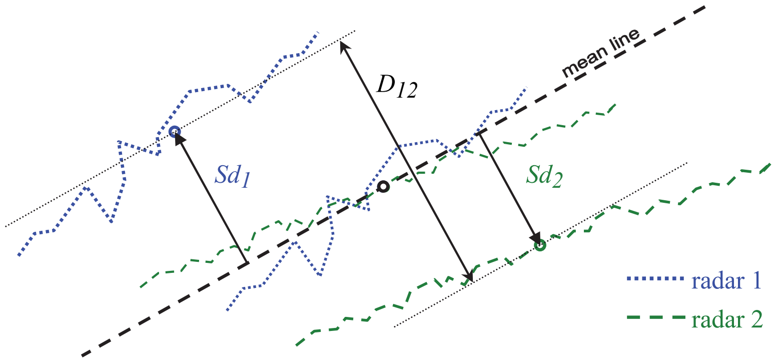


Fig. 1: Matching the track parts by using the principal components method

All radar tracks on the considered part of motion are taken into calculations. The geometric parameters of matching the tracks are determined over the totality of all radars. It allows one to obtain constructively mutually agreed parameters of matching the tracks. In practical calculations, to obtain the complete picture of situation under consideration, the overlapping (in observation time) parts of the aircraft motion are taken with measurements from all radars participating in observation.

The above described implementation of the algorithm for geometric matching of the tracks from radars (of one aircraft) works on "sole" fragments of motion. The found values D_{ij} of relative deviations of tracks are interpreted as the shifts along the normal $(\sin A_c, -\cos A_c)$. The values $\{D_{ij}, A_c\}$ obtained over the all processed fragments of motion are used for finding constant corrections to radar measurements in azimuth and in range. The totality of "actual" difference vectors of the form $D_{ij}\cdot(\sin A_c, -\cos A_c)$ is denoted by $\{(x_{Fk}, z_{Fk})\}$. On the other hand, using the model of constant systematic errors in azimuth and range, we obtain another (model) field of difference vectors, *i.e.*, the relative shifts of measurements for pairs of radars. Denote the totality of these vectors by $\{(x_{Rk}, z_{Rk})\}$. The functional to be optimized is related to deviation of two above-mentioned vector fields and can be written in the form

$$\sum_k (x_{Fk}^2 + z_{Fk}^2 - x_{Fk}x_{Rk} - z_{Fk}z_{Rk})^2 \to \min . \qquad (*)$$

The similar scheme was considered in [2]. There, the data on the actual deviation of tracks were also represented in the form of a vector field and the functional to be optimized was constructed over the field of differences as follows: $\sum((x_{Rk} - x_{Fk})^2 + (z_{Rk} - z_{Fk})^2) \to \min$. In that case, the operation of geometric matching was performed over all tracks from a small geographic region in a given time interval. From computational point of view, this was rather expensive. Moreover, it was required to fulfill special conditions related to traffic density such as the presence of intersecting tracks or tight turns. The new algorithms suggested can work in the ATC zones of responsibility with low density traffic, without obligatory fulfilling of these conditions. Moreover, computational time is crucially reduced.

Computations over model data of radar measurements

Let us illustrate the image of the difference field of shifts (stipulated by the systematic errors in azimuth Δ^{α} and range Δ^{d}) of the radar measurements. Consider two model versions (Fig. 2) for a pair of radars with the following values of systematic errors:

Version	radar No. 1		radar No. 2	
	Δ_1^{α}	Δ_1^{d}	Δ_2^{α}	Δ_2^{d}
No. 1	+0.2°	500 m	+0.2°	500 m
No. 2	+0.2°	500 m	–0.2°	0 m

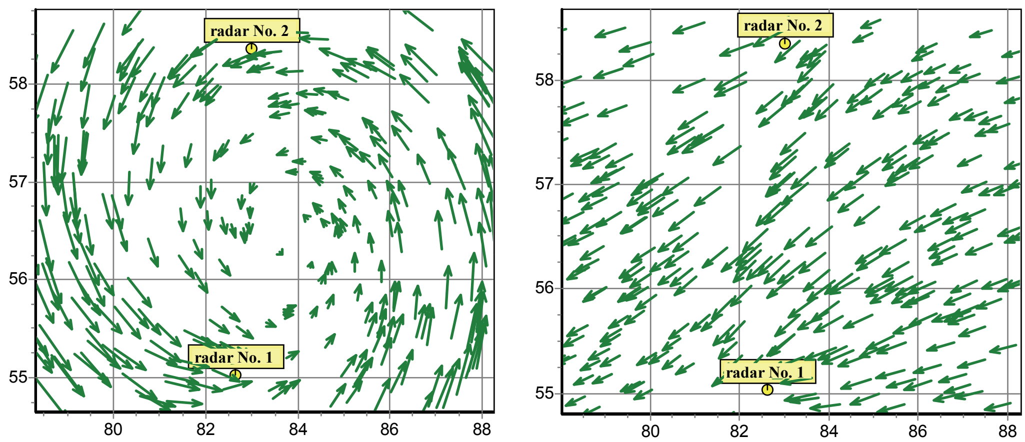


Fig. 2. Model difference shifts of the radars measurements;
version 1 (left); version 2 (right); axes: degrees of longitude and latitude

For testing the suggested method and corresponding software, the following model data of air observation in some ATC zone were simulated (Fig. 3). The data corresponds to one hour observation interval. Here, two fragments of aircraft motions are presented: before taking into account systematic radar errors (upper part) and after matching the tracks (below). Eight radars (of total twelve ones participating in observation) are marked for which the measurements are formed. The measurements contain information on azimuth, slant range, altitude of aircrafts. In measurements of several radars, the systematic errors are simulated in azimuth (between 1° – 5°) and in range (of size up to 1 kilometer).

To minimize the functional of the form (∗), the coordinate-wise descent method was used [3]. The recovery accuracy of the systematic errors in azimuth is about 0.01°. The systematic errors in range were estimated with worse accuracy. It needed up to three iterations of the method to obtain accuracy about 30 meters. In the lower part of Fig. 3, the aircraft tracks are presented. They were corrected by the estimates of the radar systematic errors in azimuth/range taken with the minus sign. It is seen that the character of the tracks matching w.r.t. the geometric coordinates has been essentially enhanced after considering the systematic errors of measurements.

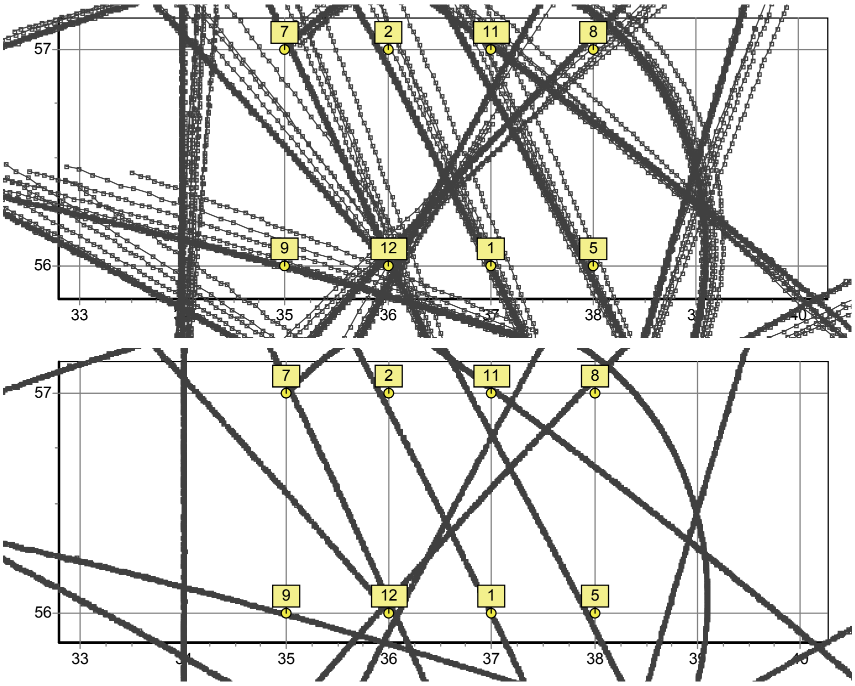
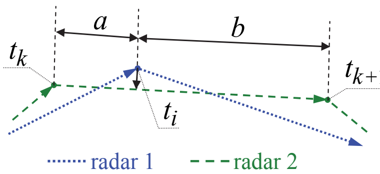


Fig. 3. Model trajectory information before (upper) and after (lower) matching

So, the radar systematic errors in azimuth and range are taken into account, *i.e.*, the matching of the tracks in the geometric coordinates has been fulfilled. Further, estimation of mutual inconsistency in time between measurements of various radar pairs is implemented. Desired estimates are calculated as an average divergence in time for the collections of the track points (Fig. 4) in the form $\left(\sum\Delta t_i - \sum\Delta t_k\right)/N$. Here, N is the total number of values Δt_i and Δt_k. The formula for Δt_i is given in the figure, where t_i are the instants of measurements from radar 1 and t_k are the instants of measurements from radar 2. The values Δt_k are calculated similarly to computations of the values Δt_i, *i.e.*, over the totality of the second radar measurements w.r.t. the measurements of the first one. The piecewise–linear approximation of radar tracks is used. As a result, the relative values of corrections in time are obtained.



$$\Delta t_i = [\, t_i - t_k - (t_{k+1} - t_k)\cdot a/(a+b)]$$

Fig. 4. Estimation of the deviation of the radar measurements in time

Conclusions

The main advantage of the method suggested for the geometric matching of tracks consists in obtaining estimates for the systematic errors of geometric positions of measurements *independently* of the time errors in the radar measurements. Corresponding additive corrections to the radar measurements are calculated using the relative parameters of the geometric matching of aircraft tracks parts. In frames of the used model of errors, the individual constant corrections in azimuth and range are found for each radar. The results of computations are given demonstrating the capabilities of the method suggested. Simulated data of air traffic observation were used.

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